Designing Instruction with the Components of Numeracy in Mind

by Lynda Ginsburg

I need to replace my refrigerator. Among other things, I will have to determine the exact size of the space available for the refrigerator because I have a pretty small kitchen, I will have to comparison shop for the appropriate combination of features and price, and I will have to decide among alternative methods of paying for the refrigerator. To accomplish the task of replacing my refrigerator, I will need:

1. a conceptual understanding of measurement (including fractions), percent (for discounts, interest), and volume (to compare features of different refrigerators),

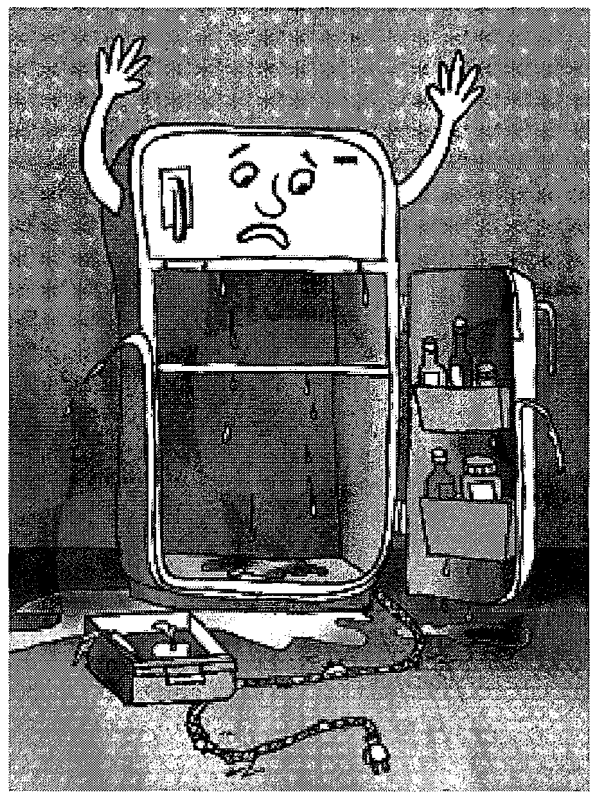
2. to reason about particular situation, identifying the demands of the task (determine fit, pay the least for the most value, minimize interest payments), and then determining which strategies will be useful (spatial reasoning, measuring dimensions, comparing different discounts and rebates, charting different financing options with different interest rates)

3. to formulate mathematical problems given the situation and figure out how to solve them,

4. to perform necessary calculations and measurements (and decide when exact answers are needed, when estimates will suffice, if numbers should be rounded up, etc.),

5. a willingness to undertake this task; a belief that mathematics is a problem solving tool not just a school subject; a belief that I can manage to do this myself; and knowing that if I get stuck, I can rethink and still succeed.

Without developing all of these cognitive and affective processes, knowing the mathematical content, and understanding the features of the context, I will not be prepared to deal with this situation in a powerful, competent way. The specific content knowledge needed for this task as well as others, along with the five processes described, enable a person to handle situations effectively. Numeracy comprises much more than arithmetic skills (had I known how to calculate a percent of a quantity, that would hardly have been sufficient); it requires the coming together of different kinds of knowledge and competence. The knowledge of context and content along with an ability to use the cognitive and affective processes are the components of numeracy. For a description of the components, see the box on page 15; for the paper in which the components were originally proposed, see: www.ncsall.net/fileadmin/resources/research/op_numeracy.pdf.



Adult educators are charged with helping learners acquire the numeracy skills to function competently and proficiently. This means attending to *all* of the components of numeracy. Questions to consider are: How should educators go about helping adult learners progress towards becoming numerate? What instructional strategies are most likely to pay off for learners given their goals for themselves? What decisions do educators need to consider about how to organize their classes, what to teach, and how to teach it? Without addressing all of the components of numeracy, while addressing these questions, we are limiting what our learners will be able to do with what they are learning.

Examining "Risky Practices" in Numeracy Instruction

It is worthwhile to stand back and, through the lens of the components of numeracy, examine five instructional practices that can be observed in many adult education programs. Each of these practices has useful and beneficial elements and reflects educators' sincere expectations that the practices will contribute to learners' success. Yet, each practice also is risky in that it narrowly defines the numeracy terrain and severely limits adult learners' opportunities to become numerate in a competent and functional way. Our primary goal as numeracy educators should be to consider how to create a balance among multiple goals, practices, and outcomes.

Risk #1. Primarily emphasizing calculation skills

Probably the most common instructional strategy found in adult education settings is a strong emphasis on mastery of arithmetic calculation skills. This approach is clean and easy: identify the arithmetic skill that the learner is "lacking," provide workbooks, start at the appropriate page and monitor the learner as he or she completes practice on consecutive topics. Generally, the particular operation (such as subtract one fraction from another) is clearly specified and modeled for the learner. The goal is for the learner to gain procedural fluency and be able to calculate accurately, efficiently, and quickly. Teachers perceive that this goal is closely aligned with some existing assessments such as the mathematics computation test of the Test of Adult Basic Education (TABE).

While learners do progress from topic to topic, this type of instruction is often not very effective in helping learners make meaning of what they are doing. Most math teachers have heard learners say when confronted with a contrived word problem with fractions, "I know how to do this. Just tell me if I should multiply or divide." Similarly, learners say, "I know the

Components and Subcomponents of Numeracy

CONTEXT - the use and purpose for which an adult takes on a task with mathematical demands

- **Family or Personal**—as a parent, household manager, consumer, financial and health-care decision maker, and hobbyist
- **Workplace**—as a worker able to perform tasks on the job and to be prepared to adapt to new employment demands
- **Further Learning**—as one interested in the more formal aspects of mathematics necessary for further education or training
- **Community**—as a citizen making interpretations of social situations with mathematical aspects such as the environment, crime and politics

CONTENT - the mathematical knowledge that is necessary for the tasks confronted

- **Number and Operation Sense**—a sense of how numbers and operations work and how they relate to the world situations that they represent
- **Patterns, Functions and Algebra**—an ability to analyze relationships and change among quantities, generalize and represent them in different ways, and develop solution methods based on the properties of numbers, operations and equations
- **Measurement and Shape**—knowledge of the attributes of shapes, how to estimate and/or determine the measure of these attributes directly or indirectly, and how to reason spatially
- **Data, Statistics and Probability**—the ability to describe populations, deal with uncertainty, assess claims, and make decisions thoughtfully

COGNITIVE AND AFFECTIVE-the processes that enable an individual to solve problems and, thereby, link the content and the context

- **Conceptual Understanding**—an integrated and functional grasp of mathematical ideas
- **Adaptive Reasoning**—the capacity to think logically about the relationships among concepts and situations
- **Strategic Competence**—the ability to formulate mathematical problems, represent them, and solve them
- **Procedural Fluency**—the ability to perform calculations efficiently and accurately by using paper and pencil procedures, mental mathematics, estimation techniques, and technological aids
- **Productive Disposition**—the beliefs, attitudes, and emotions that contribute to a person's ability and willingness to engage, use, and persevere in mathematical thinking and learning or in activities with numeracy aspects

From Ginsburg, L., Manly, M., & Schmitt, M. J. (2006). *The Components of Numeracy* [NCSALL Occasional Paper]. Cambridge, MA: National Center for Study of Adult Learning and Literacy. Available at www.ncsall.net/fileadmin/resources/research/op_numeracy.pdf steps, but I can't remember which I do first." And what teacher has not seen the strange results of a learner's computation with a misplaced decimal point, because the learner has not stopped to consider if the solution "makes sense"? Extensive practice with procedures and focusing on attaining the one right answer, does not necessarily help learners develop either number sense, estimation skills, or the reasoning skills that should be used to monitor computation and also contribute to sense making and everyday mathematics activity.

There is little reason to assume learners will be able to apply their computational procedures in productive ways if the procedures have only been practiced in isolation as drills on worksheets, workbooks, or computer programs. Without also developing learners' conceptual understanding of the meaning of the procedures and of the relationships among them, as well as providing practice seeing how the procedures are applied in real life situations, learners have only learned to do what a calculator can do efficiently and have only been prepared for a computation assessment. Seeing the connections between operations on fractions and actually measuring wood or material for a home project, or deciding whether to use a "$10 off" rather than a "30% off" coupon, requires more than computational skill. We should not assume that learners will be making the connections themselves and will know when and how to apply their computational skills.

Risk #2. Focusing on the language aspects of word problems

Most workbooks include word problems, meant to provide examples of real world applications of computational procedures. The word problems usually follow sets of computational exercises that provide practice on the procedures needed for the word problems. Sometimes, the procedures needed to solve word problems are less obvious to learners and they must decide what computation to perform.

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Some teachers look upon the word problems as a special kind of task to master: one requiring a particular set of problem solving skills and strategies. However, rather than focusing on the mathematical relationships among the elements of the problem situation described, the instructional focus is on the language clues within the word problem. Examples of this include pointing to words and phrases such as "in all," "all together," "greater than," "difference," or "each." While the words in the problem obviously contribute to one's understanding of the situation, focusing on particular words as guides to computational procedures does not promote numeracy understanding, real problem solving, or growth in being able to apply learning in real situations. Rarely do problems in everyday or work life contain "clue words". More often than not, they are situations that need to be understood, represented, and addressed with mathematical reasoning and then computation.

Risk #3. Attempting to dissipate math anxiety

Many adults returning to school fear having to deal with numbers or mathematics, especially in a school-like context. They feel that they are unable to learn what they need to know, often basing these feelings on past educational experiences, messages they have received from teachers and family members, and a cultural assumption that mathematics is hard for anyone who is not a "math person" with innate math talent that allows math learning to come easily.

Adult educators want to empower their learners to see themselves as capable and competent. They strive to create a learning environment that is safe. Some, however, assume that to be safe, a learner should not have to struggle, be wrong, or be challenged. They break mathematical content into very small, discrete pieces that learners can master without difficulty. This approach reinforces learners' learned helplessness.

In reality, for adults to function in a competent way in numeracy situations in and out of school, they need to develop a productive disposition which involves more than simply not being anxious about math. For example, encountering frustration when not being immediately able to solve an equation, find a computational error, or figure out how to find the maximum area a fence can enclose is normal. Such frustration is not a symptom of mathematical inability but is rather a part of the problem solving process. Everyone goes down blind alleys and hits walls when tasks are hard or seem complex to them. A good way to deal with this frustration is to recognize its basis and to have a repertoire of strategies available to use to attack the problem. These might include simplifying the problem by substituting small or round numbers for the numbers in the problem or by focusing attention on one aspect of the problem at a time. Other strategies include working backwards, drawing a diagram or making a table, or talking through a situation. Learners need help to build such a repertoire and chances to practice multiple strategies, and they benefit from opportunities to discuss their thinking. Once learners develop such strategic approaches to problem solving, they are less likely to feel anxious and panicky and more likely to feel control and power.

Risk #4. Primarily dividing math content into distinct, non-overlapping topics

To manage our complex lives, we often categorize or separate what we do into stand-alone chunks. In mathematics, we frequently talk about operations with whole numbers; dealing with fractions, decimals, and percents; algebra; geometry; and statistics. And then, many teachers and most workbooks address each topic in isolation, as if one had pretty much nothing to do with the others. Learners are somehow expected to make connections among them independently, if that is a goal at all.

Indeed, much evidence indicates that assumptions established and reinforced when studying one narrow topic are easy to apply but difficult to modify when studying another topic. For example, when multiplying whole numbers, people come to assume that "multiplication always makes larger" and is best imagined as repeated addition. This assumption leads to confusion when multiplying fractions or when conceptualizing the area of a shape (Fischbein et al., 1985).

Since adults have limited time to participate in formal educational activities, narrowly focusing on only one topic with few opportunities to make connections across topics does not really benefiting them. In this instructional model, adults have few opportunities to make connections between new learning and their patchy knowledge from earlier learning. And, in the everyday world, numeracy activity is rarely defined and delimited according to school-based topics. Deciding whether to pay the rent, heat, or electricity does not fit neatly into one topic.

Risk #5. Only embedding instruction within real-life contexts

Some adult educators embed mathematics instruction within contexts and problem situations that are meaningful for their learners. Whether they are real situations or simulations, they present opportunities for learners to see and experience mathematics challenges that they might encounter outside the classroom. These contexts or situations generally are less well defined than textbook word problems, require learners to make decisions, solve a consequential problem, and present and justify a solution. They provide rich opportunities for involving learners in mathematizing, or formulating math problems from within situations, and for developing problem solving skills. Learners feel that they are engaging in real problem solving, and they are. A good example can be found in Captured Wisdom's Restaurant Problem (www.ncrtec.org/pd/cw/adultlit.htm).

Researchers have found that adolescents who learn math embedded in real world, problem-based activity often need time and help from teachers to make the mathematics explicit and pull the mathematics out of the situation for it to be accessible for use in other settings or for a standardized, decontextualized test (Boaler, 2000; Stone et al., 2005). Conceptual understanding might be developing, but it is tightly linked to the context within which it was learned and practiced.

For example, in a construction context, fractions appear in a very rich way in measurement, as does proportional reasoning in reading blueprints, yet the mathematics may be encountered within specific, practice-based procedures that do not require conceptual understanding. One can use a ruler by learning the names of the lines between inch marks (i.e., ½, 5/8, 15/16) without gaining an understanding that ½, 2/4, 4/8, 8/16 are equivalent, or being able to find the distance between 8½ inches and 12 3/8 inches. And would 8.3 inches really be a useful or meaningful representation within this context? Embedding math learning deep in a real world context has benefits for helping learners master the mathematical practices of the context and solve problems within the constraints of the context, but the instructor needs to also locate the mathematical ideas within broader contexts as well so that learners can "carry" their knowledge and skills to other places.

Instruction with All Components

No one ideal way to teach mathematics meets the needs of all learners, is within the reach of all teachers, and can be supported by all programs. However, all teachers should challenge themselves to help learners acquire the range of skills and knowledge that are necessary for proficient numeracy practice. And this should lead them to question their current practices. It also might well lead them to realize that they may need to upgrade their own depth of understanding, knowledge, and skills so that they can best meet the needs of their learners.

All Components at Once

Among the practices that adult educators might consider is addressing all the components at once, not in succession. No evidence indicates that extensive practice adding fractions, multiplying decimals, or finding percent discounts will lead to conceptual understanding of fractions, decimals, or percents; an understanding of how they are related and different from each other; or an ability to decide how they might be used when representing or solving problems.

For example, when studying fractions, consider spending time talking about the relative size of fractions, i.e., what happens when numerators increase while denominators stay the same and what happens when denominators increase while numerators stay the same or which fractions are close to 0, ½, or 1 and how to know. Discuss what it means and what happens when you multiply two fractions each less than 1, or multiply a whole number by a fraction that is less than 1 or greater than 1. Draw diagrams to represent different situations. Use rulers and tape measures to add or subtract fractional quantities. Estimate answers and discuss situations in which estimates would or would not be adequate.

Encourage and celebrate the use of alternative strategies to solve problems, represent situations (in diagrams, tables, graphs, etc.), and do calculations. "Can anyone do it another way?" is not only asking for alternative thinking but is sending the message that there is always more than one way to approach a problem or computation. Discussions of alternatives, right or wrong ones, lead to deeper understanding. I'm assuming that the classroom is a safe environment for learners, which should be the norm in adult education.

I have seen that learners pick up the mantra of "Can anyone do it another way?" trying to outdo each other to come up with as many alternative solution paths as possible. In the course of this, learners are free to opine why an alternative strategy will or will not work, which strategy makes most sense to them, or which strategy seems most efficient. They start noticing patterns and see that math can be a creative enterprise. Learning math is no longer passive, but an active endeavor. I have been amazed at something a learner said, often in the context of, "Why couldn't it be ...?" and have had to really think about whether or not a particular suggestion always worked and why. Sometimes, we end up identifying one particularly useful or clever method as "Tara's method" and the nickname remains for the rest of the year.

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Not surprisingly, different procedures are better in different situations or with different numbers. In addition, different procedures work better for different people. Why not provide a rich assortment of procedures and strategies from which learners can choose the most meaningful or "easiest" for themselves?

Talk about Meaning

Make certain that learners can talk about the meaning of what they are doing: what, why, and how. In explaining their thinking to someone else, whether to another learner or a teacher, people have to really understand what they are doing. This does not mean rattling off a sequence of steps or procedures, but rather explaining why they are doing what they are doing. Sometimes, when challenged to do this, people realize that they don't actually understand what they are doing. If people don't understand or "own" their mathematical ideas and skills, they will just lose them, just as happened to them before.

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In addition, communicating about numerical information is a necessary task in the workplace and for everyday life. Developing mathematical vocabulary, not from worksheets or from vocabulary lists, but from meaningful use is empowering.

Problem Solving Processes

In class, solve real problems that may be complex and messy and worry less about getting a "correct" solution than about the problem solving processes used. Talk about alternative strategies, the choices that could be made, and the benefits and drawbacks of each.

Most of the time, numerical information and real problems do not appear in the terse language of word problems. People need to learn how to make numerical decisions when the numbers don't come out even, and even when all the information is not clearly defined. Being a flexible problem solver means knowing what to try when the obvious doesn't work or seems to lead to a dead end. Learners need to amass a repertoire of strategies and experiences that can be available when needed.

Consider Student Goals

Consider the goals of students. For those who plan to continue their education at a community college, consider making algebra the center of instruction, building in work on other topics as needed. For those who have other aspirations, build in appropriate contextual work, building on embedded math but relating it to other concepts.

There really is not a rigid sequence of topics that must be followed. Adult learners all bring a collage of mathematical knowledge with them from earlier schooling, everyday life, work experience, and even the advertising they see around them. As educators, we should be helping them make connections among all that they already know and that which they seek to know. I believe that we should start with what they want or need to learn and fill in the gaps as they become apparent. For example, an algebra class can provide ample opportunities for discussions of fractions when considering the values that lie between plotted points on a graph. Similarly, for a group preparing to enter construction trades, measuring walls, doors, and windows (including fractions or decimal measurements), using proportional reasoning for drawing and also reading blueprints, and exploring geometry strategies for ensuring that walls are perpendicular are both useful skills and mathematical concepts that should be understood and examined.

Number Sense and Estimation Skills

Develop number sense and estimation skills in addition to procedural (computational) fluency by exploring the arithmetic processes rather than solely focusing on getting correct answers. The advent of calculators makes these skills even more important for monitoring accuracy.

Often, actual computation is cumbersome and an estimate is sufficient for many real world situations. This is even true for eliminating obviously incorrect answers on standardized tests. There are strategies that people use to estimate, including focusing on round, friendly numbers while attending to the order of magnitude (consider 50 rather than 53, 14,000 rather than 13,789, etc.) and using body parts for estimating measurements. Most people seem to develop their own estimation strategies, whether for use at the store or when determining a tip in a restaurant. Why not make these mental processes apparent to learners so they can use them without having to first develop them themselves? Most learners do have some estimation strategies that they use; however, they often don't bring them into class, wrongly perceiving that in math class one must only find the one, accurate answer (even if it makes no sense).

Other times, mental math procedures are easy and accurate. Students are often amazed to find that they can do most percent problems mentally by decomposing numbers. Virtually every adult learner knows that 50 percent of a number means half of it (Ginsburg, et al., 1995), and through discussion and experimentation most come to realize that it is easy to know how much 10 percent of any number is without having to write the calculation (I have been amazed at how many people leave adult basic education without knowing this). Then, many percent calculations are possible (60% is 50% + 10%; 15% is 10% + half of 10%; etc.). Adult learners feel empowered when they are able to do such feats mentally, and they are not doing meaningless tricks that will easily be forgotten.

Build Productive Dispositions

Build productive dispositions towards mathematics and towards problem solving by demystifying all the processes and procedures and by sharing reasoning strategies. Productive dispositions refer to a learner's willingness to engage, use, and persevere in mathematical thinking and learning or in activities with numeracy aspects. Emphasizing sense-making is much more likely to help learners develop confidence than practice with procedural routines that might easily be forgotten or remembered inaccurately. Through reasoning about mathematical tasks, practicing solving problems and engaging in opportunities to see that everyone (including teachers) goes down wrong solution paths at times, learners can feel satisfaction when it "works" and better manage frustration when it doesn't. But first, they need productive tools and experiences. They gradually replace inhibiting fatalistic beliefs such as "I am not a math person" with a willingness to persevere and an expectation that meaning can be found.

Group Work

Encourage group problem solving. Multiple heads are better than one and having to justify reasoning to others helps pinpoint areas of understanding and of confusion. It is also more fun. Most of us find comfort and help when we work together. Many adult education classes develop a sense of community, with learners providing emotional and intellectual support for each other. Yet, many times, when it comes time for math, learners retreat to individualized, solo work. Collaborative work promotes discussions of alternative strategies and makes necessary the need to explain and justify ideas and solution paths. Mathematics then becomes an engaging, interactive learning process, with the teacher not necessarily the source of all information.

Ask Yourself...

All teachers can benefit from standing back and reflecting on whether the instruction being provided really helps learners become numerately competent. It is difficult to completely change deeply embedded instructional patterns, but it is easy to be curious and ask, "What will happen if I ..." noting how changes affect learners' skills, knowledge, understanding, and dispositions. This approach asks more of teachers but potentially improves outcomes for learners. If competent performance in numeracy is really a goal for our learners, then they must have opportunities to develop all of the components of numeracy and also have opportunities to put it all together.

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Lynda Ginsburg is a senior researcher for mathematics education at Rutgers University, New Brunswick, NJ. She is interested in how adults learn math when they return to school and how they work with their children around math. She has taught math at the high school, adult, and developmental math levels and has a doctorate in math education from the University of Wisconsin-Milwaukee. ❖